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A NOTE ON WEIGHTED HOMOGENEOUS SICIAK-ZAHARYUTA EXTREMAL FUNCTIONS

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Abstract. We prove that for any given upper semicontinuous function φ on an open subset E of $\mathbb{C}^n \setminus \{0\}$, such that the complex cone generated by E minus the origin is connected, the homogeneous Siciak-Zaharyuta function with the weight φ on E, can be represented as an envelope of a disc functional.

Introduction. Let $\mathcal{L}$ denote the Lelong class on $\mathbb{C}^n$ and $\mathcal{L}^h$ the subclass of functions u which are *logarithmically homogeneous.* Let $\varphi\colon E \to \overline{\mathbb{R}}$ be a function on a subset E of $\mathbb{C}^n$ taking values in the extended real line $\overline{\mathbb{R}}$. The *Siciak-Zaharyuta extremal function* $V_{E,\varphi}$ *with weight* φ is defined by

$$V_{E,\varphi} = \sup\{u \in \mathcal{L}\,;\, u|E \le \varphi\}.$$

The *homogeneous Siciak-Zaharyuta extremal function* $V^h_{E,\varphi}$ *with weight* φ is defined similarly with $\mathcal{L}^h$ in the role of $\mathcal{L}$. In the special case when $\varphi = 0$ we only write V_E (and V^h_E) and we call this function the (*homogeneous*) *Siciak-Zaharyuta extremal function for the set* E. The function V_E (V^h_E) is also called the (*homogeneous*) *pluricomplex Green function for* E *with pole at infinity.*

Theorem 1. *Let* $\varphi\colon E \to \mathbb{R} \cup \{-\infty\}$ *be an upper semicontinuous function on an open subset* E *of* $\mathbb{C}^n \setminus \{0\}$. *Assume that there exists a function in* $\mathcal{L}^h$ *dominated by* φ *on* E. *Then the largest logarithmically homogeneous function* $\mathbb{C}E \to \mathbb{R} \cup \{-\infty\}$ *dominated by* φ *on* E *is upper semicontinuous on* $\mathbb{C}^*E$ *and it is of the form* $\log \varrho_{E,\varphi}$, *where*

$$\varrho_{E,\varphi}(z) = \inf\{|\lambda| e^{\varphi(z/\lambda)}\,;\, \lambda \in \mathbb{C}^*,\ z/\lambda \in E\}, \qquad z \in \mathbb{C}^*E. \tag{1}$$

If $\mathbb{C}^*E$ *is connected, then for every* $z \in \mathbb{C}^n$

$$\begin{aligned} V^h_{E,\varphi}(z) = \inf\Big\{ \int_{\mathbb{T}} \log \varrho_{E,\varphi}(f_1, \dots, f_n)\, d\sigma\,;\, f \in \mathcal{O}(\overline{\mathbb{D}}, \mathbb{P}^n),\ f = [f_0 : \cdots : f_n], \\ f(\mathbb{T}) \subset \mathbb{C}^*E,\ f_0(0) = 1,\ (f_1(0), \dots, f_n(0)) = z.\Big\} \end{aligned} \tag{2}$$

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If $\mathbb{C}E = \mathbb{C}^n$*, then for every* $z \in \mathbb{C}^n$

$$V^h_{E,\varphi}(z) = \inf\Big\{ \int_{\mathbb{T}} \log \varrho_{E,\varphi} \circ f \, d\sigma \,;\, f \in \mathcal{O}(\overline{\mathbb{D}}, \mathbb{C}^n),\ f(0) = z \Big\}. \tag{3}$$

A disc envelope formula is a formula where the values of a function F defined on a complex space X with values on the extended real line $\overline{\mathbb{R}}$ are given as $F(z) = \inf\{H(f)\,;\, f \in \mathcal{B}(z)\}$, where H is *disc functional*, i.e., a function defined on some subset $\mathcal{A}$ of $\mathcal{O}(\mathbb{D}, X)$, the set of *analytic discs* in X, with values on $\overline{\mathbb{R}}$, $\mathcal{B}$ is a subclass of $\mathcal{A}$, and $\mathcal{B}(z)$ consists of all of $f \in \mathcal{B}$ with *center* $z = f(0)$.

The formula (2) is an example of a disc envelope formula, where $\mathcal{A}$ consists of all closed analytic discs with value in the projective space, i.e., elements f in $\mathcal{O}(\overline{\mathbb{D}}, \mathbb{P}^n)$ which map the unit circle $\mathbb{T}$ into $\mathbb{C}^*E$, $H(f)$ is the integral, and $\mathcal{B}$ is the subset of $\mathcal{A}$ consisting of discs with $f_0(0) = 1$. We identify a point $[1 : z] \in \mathbb{P}^n$ with the point $z \in \mathbb{C}^n$.

For general information on the Siciak-Zaharyuta extremal function see Siciak [8, 9, 10, 11, 12] and Zaharyuta [13]. The first disc envelope formula for V_E was proved by Lempert in the case when E is an open convex subset of $\mathbb{C}^n$ with real analytic boundary. (The proof is given in Momm [5, Appendix].) Lárusson and Sigurdsson [2] proved disc envelope formulas for V_E for open connected subsets E of $\mathbb{C}^n$. Magnússon and Sigurdsson [4] generalized this result and obtained a disc formula for $V_{E,\varphi}$ in the case when φ is an upper semicontinuous function on an open connected subset E of $\mathbb{C}^n$. Drinovec Drnovšek and Forstnerič [1] proved disc envelope formulas for V_E for open subsets E of an irreducible and locally irreducible algebraic subvariety of $\mathbb{C}^n$. Recently, Magnússon [3] established disc envelope formulas for the global extremal function in projective space.

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Notation. Let $\mathbb{D}$ denote the unit disc in $\mathbb{C}$, $\mathbb{T}$ the unit circle, and σ the arc length measure on $\mathbb{T}$ normalized to 1. An analytic disc is a holomorphic map $f \colon \mathbb{D} \to X$, where X is some complex space. We let $\mathcal{O}(\mathbb{D}, X)$ denote the set of all analytic discs. We say that the disc is closed if it extends as a holomorphic map to some neighbourhood of the closed unit disc $\overline{\mathbb{D}}$ with values in X and we let $\mathcal{O}(\overline{\mathbb{D}}, X)$ denote the set of all closed analytic discs in X. The point $z = f(0) \in X$ is called the center of f.

For a subset X of $\mathbb{C}^n$ we let $\mathcal{USC}(X)$ denote the set of all upper semicontinuous functions on X, and for open subset U of $\mathbb{C}^n$ we denote by $\mathcal{PSH}(U)$ the set of all plurisubharmonic functions on U. The Lelong class $\mathcal{L}$ consists of all $u \in \mathcal{PSH}(\mathbb{C}^n)$ such that $u - \log^+|\cdot|$ is bounded above and $\mathcal{L}^h$ is the

subclass of all logarithmically homogeneous functions, i.e., functions satisfying $u(\lambda z) = u(z) + \log|\lambda|$ for $\lambda \in \mathbb{C}^*$ and $z \in \mathbb{C}^n$. Observe that every such function takes the value $-\infty$ at the origin. For every subset E of $\mathbb{C}^n$ we set $\mathbb{C}E = \{\lambda z\,;\, \lambda \in \mathbb{C}, z \in E\}$, $\mathbb{C}^*E = \{\lambda z\,;\, \lambda \in \mathbb{C}^*, z \in E\}$ and we call $\mathbb{C}E$ the complex cone generated by E. Note that complex cones are suitable sets for the domains of definition of logarithmically homogeneous functions.

Let $\mathbb{P}^n$ denote the n-dimensional projective space, $\pi\colon \mathbb{C}^{n+1} \setminus \{0\} \to \mathbb{P}^n$ the natural projection, $(z_0, \ldots, z_n) \mapsto [z_0 : \cdots : z_n]$, and identify $\mathbb{C}^n$ with the subset of all $[z_0 : \cdots : z_n]$ with $z_0 \neq 0$ and, in particular, the point $z \in \mathbb{C}^n$ with $[1 : z] \in \mathbb{P}^n$. *The hyperplane at infinity* is $H_\infty = \pi\big(Z_0 \setminus \{0\}\big)$, where Z_0 is the hyperplane in $\mathbb{C}^{n+1}$ defined by the equation $z_0 = 0$. Then $\mathbb{P}^n = \mathbb{C}^n \cup H_\infty$.

Review of a few results. Assume that $\psi\colon X \to \mathbb{R} \cup \{-\infty\}$ is a measurable function on a subset X of $\mathbb{C}^n$, such that there is $u \in \mathcal{L}$ satisfying $u|X \leq \psi$. It is an easy observation that a function $u \in \mathcal{PSH}(\mathbb{C}^n)$ is in $\mathcal{L}$ if and only if the function

$$(z_0, \ldots, z_n) \mapsto u(z_1/z_0, \ldots, z_n/z_0) + \log|z_0|$$

extends as a plurisubharmonic function from $\mathbb{C}^{n+1} \setminus Z_0$ to $\mathbb{C}^{n+1} \setminus \{0\}$. Let v denote this extension. Take $f = [f_0 : \cdots : f_n] \in \mathcal{O}(\overline{\mathbb{D}}, \mathbb{P}^n)$ with $f_0(0) = 1$, $(f_1(0), \ldots, f_n(0)) = z$, satisfying $f(\mathbb{T}) \subset X$, and define $\tilde{f} = (f_0, \ldots, f_n) \in \mathcal{O}(\overline{\mathbb{D}}, \mathbb{C}^{n+1} \setminus \{0\})$. Then the subaverage property of $v \circ \tilde{f}$ and the Riesz representation formula applied to $\log|f_0|$ give (see [4, p. 243])

$$\begin{aligned} u(z) &= u(f_1(0), \ldots, f_n(0)) + \log|f_0(0)| = v \circ \tilde{f}(0) \\ &\leq \int_{\mathbb{T}} u(f_1/f_0, \ldots, f_n/f_0)\, d\sigma + \int_{\mathbb{T}} \log|f_0|\, d\sigma \\ &\leq \int_{\mathbb{T}} \psi(f_1/f_0, \ldots, f_n/f_0)\, d\sigma - \sum_{a \in f^{-1}(H_\infty)} m_{f_0}(a) \log|a|. \end{aligned}$$

For an open connected $X \subset \mathbb{C}^n$ and $\psi \in \mathcal{USC}(X)$, Magnússon and Sigurdsson [4, Theorem 2] proved that for every $z \in \mathbb{C}^n$

$$\begin{aligned} V_{X,\psi}(z) = \inf\big\{ &- \sum_{a \in f^{-1}(H_\infty)} m_{f_0}(a) \log|a| + \int_{\mathbb{T}} \psi(f_1/f_0, \ldots, f_n/f_0)\, d\sigma\,; \\ &f \in \mathcal{O}(\overline{\mathbb{D}}, \mathbb{P}^n),\ f(\mathbb{T}) \subset X,\ f_0(0) = 1,\ (f_1(0), \ldots, f_n(0)) = z\big\}. \end{aligned} \tag{4}$$

Our main result, Theorem 1, will follow from this formula and the following

Proposition 2. *Let $\varphi\colon E \to \mathbb{R} \cup \{-\infty\}$ be a function on a subset $E \subset \mathbb{C}^n \setminus \{0\}$ such that there exists $u \in \mathcal{L}^h$ satisfying $u|E \leq \varphi$. Let $\tilde{\varphi}\colon \mathbb{C}E \to \mathbb{R} \cup \{-\infty\}$ be the supremum of all logarithmically homogeneous functions on $\mathbb{C}E$ dominated by φ on E. Then the following hold:*

(i) *$\tilde\varphi$ is logarithmically homogeneous on $\mathbb{C}E$ and for every $z \in \mathbb{C}^*E$*

$$\tilde\varphi(z) = \inf\{\varphi(\lambda z) - \log|\lambda|\,;\ \lambda \in \mathbb{C}^* \text{ and } \lambda z \in E\}, \tag{5}$$

(ii) $V^h_{E,\varphi} = V^h_{E,\tilde\varphi} = V^h_{\mathbb{C}^*E,\tilde\varphi}$.

If, in addition to the above, $\mathbb{C}^*E$ is nonpluripolar and $\varphi \in \mathcal{USC}(E)$ then

(iii) *$\tilde\varphi \in \mathcal{USC}(\mathbb{C}^*E)$ and $V^h_{E,\varphi} = V_{\mathbb{C}^*E,\tilde\varphi}$,*

(iv) *if $\mathbb{C}E = \mathbb{C}^n$, then $\tilde\varphi \in \mathcal{USC}(\mathbb{C}^n)$ and*

$$V^h_{E,\varphi} = \sup\{u \in \mathcal{PSH}(\mathbb{C}^n)\,;\ u \le \tilde\varphi\}.$$

Proof. (i) It is easy to see that the supremum of any family of logarithmically homogeneous functions defined on a complex cone is a logarithmically homogeneous function provided the family is bounded from above at any point of the cone. Take $z \in \mathbb{C}^*E$ and choose $\lambda \in \mathbb{C}^*$ such that $\lambda z \in E$. For any logarithmically homogeneous function u on $\mathbb{C}E$ dominated by φ on E we have

$$u(z) = u(\lambda z) - \log|\lambda| \le \varphi(\lambda z) - \log|\lambda| \tag{6}$$

which implies that the family is bounded from above at z. Since all logarithmically homogeneous functions take the value $-\infty$ at the origin the family is bounded from above at any point of the cone.

Let ψ denote the function on $\mathbb{C}^*E$ whose value at z is given by the right hand side of the equation (5). For a logarithmically homogeneous function u on $\mathbb{C}E$, dominated by φ on E, we have $u(z) \le \varphi(\lambda z) - \log|\lambda|$ for any $\lambda \in \mathbb{C}^*$ such that $\lambda z \in E$ by (6). Taking infimum over all $\lambda \in \mathbb{C}^*$ with $\lambda z \in E$ shows that $u \le \psi$ on $\mathbb{C}^*E$. Hence $\tilde\varphi \le \psi$ on $\mathbb{C}^*E$. To prove the converse inequality note that

$$\begin{aligned} \psi(\mu z) &= \inf\{\varphi(\lambda\mu z) - \log|\lambda|\,;\ \lambda \in \mathbb{C}^* \text{ and } \lambda\mu z \in E\} \\ &= \inf\{\varphi(\lambda\mu z) - \log|\lambda\mu|\,;\ \lambda \in \mathbb{C}^* \text{ and } \lambda\mu z \in E\} + \log|\mu| \\ &= \psi(z) + \log|\mu| \end{aligned} \tag{7}$$

for any $z \in \mathbb{C}^*E$ and $\mu \in \mathbb{C}^*$ thus the map ψ is logarithmically homogeneous. Since $\psi \le \varphi$ on E we get $\psi \le \tilde\varphi$.

(ii) Since $\varphi \ge \tilde\varphi$ on E and $E \subset \mathbb{C}^*E$ we have $V^h_{E,\varphi} \ge V^h_{E,\tilde\varphi} \ge V^h_{\mathbb{C}^*E,\tilde\varphi}$. For proving the two equalities we take $u \in \mathcal{L}^h$ with $u|E \le \varphi$. By (i) we obtain $u \le \tilde\varphi$ on $\mathbb{C}^*E$ which implies $V^h_{\mathbb{C}^*E,\tilde\varphi} \ge V^h_{E,\varphi}$.

(iii) To prove that $\tilde\varphi$ is upper semicontinuous take $z_0 \in \mathbb{C}^*E$ and $c > \tilde\varphi(z_0)$. We need to show that $c > \tilde\varphi(z)$ for all z in some neighbourhood U of z_0. We choose $\lambda_0 \in \mathbb{C}^*$ such that $\lambda_0 z_0 \in E$ and such that $c > \varphi(\lambda_0 z_0) - \log|\lambda_0|$. Since $\varphi \in \mathcal{USC}(E)$ there exists an open neighbourhood U of z_0 such that $\lambda_0 z \in E$ and $c > \varphi(\lambda_0 z) - \log|\lambda_0|$ for all $z \in U$. By (i) we have $c > \tilde\varphi(z)$ for all $z \in U$.

Since $\mathcal{L}^h \subset \mathcal{L}$ we have $V^h_{\mathbb{C}^*E,\tilde\varphi} \le V_{\mathbb{C}^*E,\tilde\varphi}$. For proving the opposite inequality we take $u \in \mathcal{L}$ such that $u \le \tilde\varphi$ on $\mathbb{C}^*E$. Then $u(\lambda z) - \log|\lambda| \le \tilde\varphi(\lambda z) - \log|\lambda| = \tilde\varphi(z)$ for all $z \in \mathbb{C}^*E$ and $\lambda \in \mathbb{C}^*$. Let v be the upper

semicontinous regularization of the function $\sup\{u(\lambda\cdot)-\log|\lambda|\,;\,\lambda\in\mathbb{C}^*\}$ on $\mathbb{C}^n$. We have $u\le v\le\tilde\varphi$ on $\mathbb{C}^*E$ and since $\mathbb{C}^*E$ is nonpluripolar and $\tilde\varphi$ is locally bounded above on $\mathbb{C}^*E$, we have $v\in\mathcal{L}$. A similar calculation as in (7) shows that v is logarithmically homogeneous, which proves the opposite inequality.

(iv) The fact that $\tilde\varphi$ is upper semicontinuous at 0 easily follows from the fact that $\tilde\varphi$ is bounded from above on the unit sphere and that it is logarithmically homogeneous. By (iii) we get $V^h_{E,\varphi}=V_{\mathbb{C}^*E,\tilde\varphi}$ and it is easy to see that in the case $\mathbb{C}E=\mathbb{C}^n$ the latter equals $V_{\mathbb{C}^n,\tilde\varphi}$.

Let $P_{\tilde\varphi}$ denote the function whose value at z is given by the right hand side of the equation. Since $\mathcal{L}\subset\mathcal{PSH}(\mathbb{C}^n)$ it follows $V_{\mathbb{C}^n,\tilde\varphi}\le P_{\tilde\varphi}$. To prove the opposite inequality, it is enough to show that $P_{\tilde\varphi}\in\mathcal{L}$. Since $\tilde\varphi\subset\mathcal{USC}(\mathbb{C}^n)$ it follows that $P_{\tilde\varphi}$ is the largest plurisubharmonic function on $\mathbb{C}^n$ dominated by $\tilde\varphi$. By upper semicontinuity the map $\tilde\varphi$ is bounded from above on the unit sphere in $\mathbb{C}^n$ by some constant $M\in\mathbb{R}$. Since $\tilde\varphi$ is logarithmically homogeneous we get

$$P_{\tilde\varphi}(\lambda z)\le\tilde\varphi(\lambda z)\le\log|\lambda|+M=\log|\lambda z|+M$$

for any $z\in\mathbb{C}^n$, $|z|=1$, and $\lambda\in\mathbb{C}^*$. It follows that $P_{\tilde\varphi}\in\mathcal{L}$. □

Proof of Theorem 1. By Proposition 2 the largest logarithmically homogeneous function $\tilde\varphi\colon\mathbb{C}E\to\mathbb{R}\cup\{-\infty\}$ dominated by φ on E is upper semicontinuous on $\mathbb{C}^*E$ and $\varrho_{E,\varphi}=e^{\tilde\varphi(z)}=\inf\{|\lambda|e^{\varphi(z/\lambda)}\,;\,\lambda\in\mathbb{C}^*,\ z/\lambda\in E\}$ which proves (1).

If we take $X=\mathbb{C}^*E$ and $\psi=\tilde\varphi$ in (4), then logarithmic homogeneity of $\tilde\varphi$ on $\mathbb{C}^*E$ implies that

$$\int_{\mathbb{T}}\tilde\varphi(f_1/f_0,\dots,f_n/f_0)\,d\sigma=\int_{\mathbb{T}}\tilde\varphi(f_1,\dots,f_n)\,d\sigma-\int_{\mathbb{T}}\log|f_0|\,d\sigma.$$

If $f_0(0)=1$, then the Riesz representation formula gives

$$\sum_{a\in f^{-1}(H_\infty)}m_{f_0}(a)\log|a|+\int_{\mathbb{T}}\log|f_0|\,d\sigma=0,$$

which implies that the right hand side of (4) reduces to

$$\begin{aligned}V_{\mathbb{C}^*E,\tilde\varphi}(z)=\inf\Big\{\int_{\mathbb{T}}\tilde\varphi(f_1,\dots,f_n)\,d\sigma\,;\,f\in\mathcal{O}(\overline{\mathbb{D}},\mathbb{P}^n),\\ f(\mathbb{T})\subset\mathbb{C}^*E,\ f_0(0)=1,\ (f_1(0),\dots,f_n(0))=z\Big\},\end{aligned}$$

thus (2) follows from Proposition 2 (iii).

If $\mathbb{C}E=\mathbb{C}^n$ then Proposition 2 (iv) and Poletsky theorem [6, 7] imply

$$\begin{aligned}V^h_{E,\varphi}&=\sup\{u\in\mathcal{PSH}(\mathbb{C}^n)\,;\,u\le\tilde\varphi\}\\&=\inf\Big\{\int_{\mathbb{T}}\log\varrho_{E,\varphi}\circ f\,d\sigma\,;\,f\in\mathcal{O}(\overline{\mathbb{D}},\mathbb{C}^n),\ f(0)=z\Big\}\end{aligned}$$

which proves (3). □

Observation. In the special case $\varphi = 0$ we write ϱ_E for $\varrho_{E,\varphi}$. The function ϱ_E is absolutely homogeneous of degree 1, i.e., $\varrho_E(z\zeta) = |z|\varrho_E(\zeta)$. Thus, if E is a balanced domain, i.e., $\overline{\mathbb{D}}E = E$, then ϱ_E is its Minkowski function.

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